Physics 445: Problem Set 1

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Due Tuesday, April 12, 1:30 p.m.

1. Consider a complex scalar field, whose action is invariant under a global U(1) symmetry group.

a) Find the expression of the Noether current associated with the global U(1) symmetry in terms of the complex scalar field.
b) Express this current in terms of the radial and phase fluctuations of the scalar field.

Assume that the U(1) symmetry is spontaneously broken.

$$\phi = \frac{1}{\sqrt{2}} \exp(i\zeta(x)/v)\ (v + \rho(x)) \tag{1}$$

and show that ζ becomes a properly normalized Nambu-Goldstone mode.
c) Demonstrate that the Nambu-Goldstone boson interaction terms may be written as

$$\mathcal{L}_{\rm int} \simeq \frac{1}{v}\zeta\ \partial_\mu j^\mu \tag{2}$$

where j^μ is the Noether current (In general, j^μ is the current whose charge

$$Q = \int d^3x j_0 \tag{3}$$

generates the symmetry which is spontaneously broken). The coupling is inversely proportional to the scale of symmetry breaking v.

Extra credit: Try to do the same for unitary, non-abelian symmetry groups.

2. Consider a U(1) $\times$ U(1) gauge theory with coupling constants g, g'. A complex scalar field Φ carries charge (1,-1) under the gauge group and $| < \Phi > | = v$. Compute the spectrum of gauge bosons in this theory.

Do the same for a gauge symmetry group $SU(2)$ and a scalar field in a triplet of $SU(2)$. Assuming that $Q = T_3 + Y$, and knowing that the triplet has $T_3 = (1, 0, -1)$ and at least one neutral component that acquires vacuum expectation value, how does the result depend

on the hypercharge of the multiplet ? How does the resulting spectrum compare with the Standard Model one ?

3. Demonstrate that, if $A_\mu = A^a_\mu T_a$, the transformed field

$$A'_\mu = U A_\mu U^\dagger - iU\partial_\mu U^\dagger \tag{4}$$

can also be expressed by $A'_\mu = A'^a_\mu T_a$.

Hint: Use the fact that any finite transformation may be expressed by combination of infinitesimal transformation.